Optimization and Economic Theory
Syllabus

1 Introduction.

Two mathematical topics, optimization and equilibrium, lie at the foundation of much of modern economic research. Econ 4111 provides an introduction to optimization.

Econ 4111 covers material from Econ 508 (a three week intensive course on finite dimensional optimization and related topics taught to incoming economics graduate students), Econ 511 (a course in analysis tailored to incoming economics graduate students), and Econ 503/4 (the first-year graduate core microeconomics theory courses). Obviously, I cannot compress all of these graduate courses into one undergraduate course. Instead, Econ 4111 picks and chooses topics, generally focusing on topics that are foundational. I discuss the organization of the course in greater detail below.

One aspect of the course bears emphasis: the course is mathematically rigorous in a definition-theorem-proof sense. Almost without exception, if I need a result, then I prove it, if not in class (because of time constraints), then in the notes. My experience is that students come to like this type of math, but it may be very different from what you are used to.

A related comment is that on a spectrum that has “economics” at one end and “mathematics” at the other, this course lies very close to the “mathematics” end of the spectrum. There is quite a bit of economics in the Applications part of the course, but weeks may pass early on where the material is almost entirely mathematical.

Econ 4111 is intended to make first year graduate school easier. But it won’t make graduate school easy. Graduate courses are fast paced, homework can be burdensome, the material can be challenging even if you already know the math, tests can be difficult, competition is keen, and grading is exacting.

Finally, while Econ 4111 has some overlap with Math 310, 318, 4111, the Math Department’s set theory and analysis sequence, Econ 4111 is a complement to, not a substitute for, these courses. Students applying to economics PhD programs, especially top programs, should take analysis from the Math Department. You may, however, find that this course makes the analysis sequence easier.


2 Prerequisites.

Econ 4111 assumes that you have taken a year of calculus and requires, in addition, a semester of matrix algebra and Econ 4011 (intermediate micro).

This class is intended to be self-contained other than for basic calculus and matrix algebra. And even for calculus and matrix algebra, all that I really need is fairly primitive. I assume that you can differentiate \(x^2\), \(e^x\) and \(\ln(x)\) and I assume that you are familiar with matrix multiplication (which you may have learned in “pre-calculus” in high school).

If you have have taken, or are taking, Math 310, Math 318, or Math 4111, then there will be overlap with portions of this course. But Math 310, 318 and Math 4111 are not prerequisites and a case could even be made for taking this class before, or with, rather than after, these math courses.

3 Requirements.

There will be homework, more or less weekly, quizzes, and two exams, a midterm and a final.

Starting roughly in the third week of the course, I will give a very short (10 minute) quiz, one a week, in which you must prove a result. You will have the required proof in advance, so you could just reproduce the proof from memory, although ideally you would make it your own (by adding an explanatory sentence, for example). The quiz has two purposes. The first is to give you feedback well in advance of the exams. The second is to help keep you on track with the material. The material is cumulative, a bit like learning a new language, and if you fall behind, it can be difficult to catch up again. If there are major problems with your quiz then you must do it again, in office hours or some other mutually convenient time, until you get it right.

Regarding homework, if you make a clear good faith effort, then you will get full or almost full credit at the time that I assign grades. My own solutions to homework in previous years are circulating. If you simply transcribe my answers you will be marked down completely. Similarly, even if you work closely with others (which I allow), you must still write up your own solutions. This is in your own interest: one of the main points of the homework is to give you individualized feedback to help prepare you for the grading on the exams. If you don’t have access to previous answers, or to friends who know the answers, I will (happily) supply answers if you come to office hours and ask. BUT, regardless of whether you have old answers, work with friends, or come to me, you should always try to do the homework on your own first. Get stuck, really seriously stuck, and then ask for help. Try to do the homework a few days before my office hours (over the weekend, say). If you don’t struggle with the material, you won’t learn it.

Your grade will be a weighted average of the midterm, the final, the quizzes, and
the homework. Homework will count for 15% of your grade, quizzes will count for
15%, and the two exams will each count for 35%. But I reward improvement. If
you do well on the final, then you will do well on the course even if you bombed the
midterm and even if you struggled (but tried!) with the homework and quizzes. On
the other hand, however, the final will never count for less than 35% of your grade.

The tentative date for the midterm is Wednesday, March 21 (the Wednesday
after break), in class. The final will be Thursday May 3, 8-10 am, as posted in
WUCrsL, among other places.

Homework will typically be due in class on Wednesday.

4 Texts.

4.1 The Main Text.

I have my own notes for most of this material. These are available at my website. I
update the notes regularly, to fix errors, add new material, and more generally just
to make them better.

There is no required text. For the applications section, a standard text is Mas-
Colell, Whinston and Green (1995) (MWG). But this course will focus more on tools
than applications, and so the overlap will be mostly with the appendices of MWG
rather than its body.

4.2 Supplemental Texts.

I am including the following texts by way of reference. There is no expectation that
you will look at any of them this semester. But I will have a few of them put on
reserve.

4.2.1 Graduate Microeconomic Theory.

The main competitor for Mas-Colell, Whinston and Green (1995) (MWG) is Kreps
(2012). The latter is actually just the first volume of a planned multivolume text.
The first volume covers decision theory and topics in competitive economies. Other

All of these books have their pluses and minuses. MWG is comprehensive but it
can be just too much: too much material, too concise, and too technical. It seems
to work better as a reference than as a text. So you may find the other books useful
on a first pass. Kreps (2012) is particularly good for its rigorous development of
decision theory. More generally, it is often more detailed than MWG whenever the
two overlap. Varian (1992) is good as an overview for basic competitive demand
theory. Jehle and Reny (2000) is particularly good for game theory (irrelevant,
however, for Econ 4111).
4.2.2 Mathematics for Economists.

There are a number of relatively advanced analysis for economists texts, written by economists for courses more or less like this one. These include Corbae, Stinchcombe and Zeman (2009), Ok (2007), Vohra (2005), Carter (2001), and de la Fuente (2000).

These texts are in addition to the more traditional, typically more optimization centric, math for economists texts, such as Bergin (2015), Simon and Blume (1994), Sundaram (1996), and Novshek (1993).

A number of other courses have online notes, such as Truman Bewley’s (at Yale) and notes by David Ahn, Chris Shannon, and Leo Simon (all at Berkeley). I’m not providing links because the links keep breaking.

Intriligator (2002) is older (it is a reprinting of a 1971 book) and more elementary but I find it very useful. Takayama (1985) is an encyclopedic tour of a very large number of topics in classical mathematical economics.


4.3 Other Texts.

There are a number of basic analysis texts. I like Abbott (2002). It has very few errors and very good exercises. But the focus in Abbott (2002) is mainly on $\mathbb{R}$ and on topics (such as integration) that we won’t be spending any time on. Rudin (1976) is the book used in my analysis course years ago and still seems to be the modal choice for analysis courses today.

I originally learned measure theory and functional analysis using Royden (2010) (needless to say, I used a much earlier version). We won’t be doing any measure theory, but we will be doing some functional analysis, and my notes on this owe a clear debt to Royden (2010). Another useful book on functional analysis, one written expressly for economists, is Aliprantis and Border (1999). My favorite introduction to general topology is Munkres (1999).

On convexity, the classic reference is Rockafellar (1970). It is also a classic cite for convex, finite-dimensional optimization. But you may find it difficult.

For optimization, both finite and infinite dimensional, the classic (but advanced) text is Luenberger (1969).

Finally, I generally find the Wikipedia entries on math to be quite good.

5 Outline.

1. Existence of an Optimum. The basic existence result is that an optimization problem has a solution if the objective function is continuous and the domain
is compact. To understand this statement, let alone prove it, you need to know quite a bit about metric spaces. I develop the required mathematical machinery in detail, both to handle the existence theorem and because the machinery itself has wide application. There is substantial overlap between this part of the course and Math 310 and Math 4111.

2. **Finite Dimensional Optimization.** The principal results are the Karush-Kuhn-Tucker Theorem and the Envelope Theorem. In covering this material, I cover material on vector analysis and some material on convexity, including the basic (finite dimensional) support and separation theorems. The vector analysis material will be review if you have taken Math 318. You will have seen Lagrange multipliers in Math 233 or Econ 493. Otherwise, however, this material will be largely new.

3. **Applications.** I will decide later in the semester what material to plug into this section. In the past, I have covered material drawn from basic decision theory (MWG Chapter 1), basic utility theory and competitive consumer theory (MWG Chapters 1-3), competitive producer theory (MWG Chapter 5), efficiency and competitive equilibrium (MWG Chapter 10, 16), and simple agency and screening models (MWG Chapter 14). We cannot do all of this. There may be some overlap between some of this material and Econ 4011, but most of it, and perhaps all of it, will be new to you.

4. **Dynamic Optimization.** I discuss optimization for both finite horizon and infinite horizon dynamic optimization problems. This subject merits a course on its own, but because of time constraints, I will provide only an introduction. This material is likely to be almost entirely new to you.

6 **Help.**

My office hours are Tuesdays 1:30-4 PM in Seigle 386. To arrange a different time, e-mail me at nachbar@wustl.edu.

I stress that I want to meet with you if you are having difficulty, or if you simply want to talk about some aspect of the course (including how to do the homework) or about anything else (e.g., applying to graduate school).

The TA for the course will be Patrick Butenhoffer.

7 **Information from the Provost’s office.**

**Accommodations based upon sexual assault:** The University is committed to offering reasonable academic accommodations to students who are victims of sexual assault. Students are eligible for accommodation regardless of whether they seek
criminal or disciplinary action. Depending on the specific nature of the allegation, such measures may include but are not limited to: implementation of a no-contact order, course/classroom assignment changes, and other academic support services and accommodations. If you need to request such accommodations, please direct your request to Kim Webb (kim_webb@wustl.edu), Director of the Relationship and Sexual Violence Prevention Center. Ms. Webb is a confidential resource; however, requests for accommodations will be shared with the appropriate University administration and faculty. The University will maintain as confidential any accommodations or protective measures provided to an individual student so long as it does not impair the ability to provide such measures.

If a student comes to me to discuss or disclose an instance of sexual assault, sex discrimination, sexual harassment, dating violence, domestic violence or stalking, or if I otherwise observe or become aware of such an allegation, I will keep the information as private as I can, but as a faculty member of Washington University, I am required to immediately report it to my Department Chair or Dean or directly to Ms. Jessica Kennedy, the University’s Title IX Director. If you would like to speak with directly Ms. Kennedy directly, she can be reached at (314) 935-3118, jwkennedy@wustl.edu, or by visiting the Title IX office in Umphath Hall. Additionally, you can report incidents or complaints to the Office of Student Conduct and Community Standards or by contacting WUPD at (314) 935-5555 or your local law enforcement agency. See: Title IX.

You can also speak confidentially and learn more about available resources at the Relationship and Sexual Violence Prevention Center by calling (314) 935-8761 or visiting the 4th floor of Seigle Hall. See: Relationship and Sexual Violence Prevention Center.

Bias Reporting: The University has a process through which students, faculty, staff and community members who have experienced or witnessed incidents of bias, prejudice or discrimination against a student can report their experiences to the University’s Bias Report and Support System (BRSS) team. See brss.wustl.edu.

Mental Health: Mental Health Services’ professional staff members work with students to resolve personal and interpersonal difficulties, many of which can affect the academic experience. These include conflicts with or worry about friends or family, concerns about eating or drinking patterns, and feelings of anxiety and depression. See: shs.wustl.edu/MentalHealth.

References


