Econ 511: Syllabus

1 Introduction

A bit less than 2/3 of Econ 511 is an introduction to topology and analysis. This material is basic to much of economic and econometric theory and is a prerequisite for most of the advanced mathematics you might wish to acquire.

The remainder of Econ 511 comprises modules on mathematics topics that are relevant for economics. In recent years, modules have been on convexity, fixed point theorems, and monotone comparative statics.

Econ 511 is not intended as a substitute for taking courses directly from the Math Department. In particular, it is not a substitute for taking the basic analysis sequence, Math 4111 and 4121. Even after taking Econ 511, you will find these courses challenging (although less challenging than if you had not taken Econ 511).

If you have already taken a course in analysis, some of Econ 511 will be review, and you can elect not to take the course. But you may still find Econ 511 helpful because it focuses on material that is especially useful for economics. In addition, some of the modules cover material that is not contained in standard undergraduate mathematics classes.

If you choose not to take Econ 511, then you must take a substitute course. Typically, the substitute is an upper level course from the Math Department. You can discuss this with me and with the Director of Graduate Studies.

An objective of Econ 511 is to help you develop the ability to read and write mathematics. It takes years to develop this ability, so it is important to start now if you haven’t already. In contrast to Econ 508, the mini course you just completed, Econ 511 is not intended to be of immediate use to you in your other courses.

2 Requirements

You will have a weekly homework assignment. Hand in homework to Carissa in the Department office by 5pm on the due date. You should alert me if the volume of homework becomes so burdensome that it leaves you with too little time for your core economics classes. This course should not become a main focus of your efforts.

On homework, you are encouraged to work together. But you must try to do the problems yourself first. I expect those of you who are mathematically more experienced to help those of you who are less so. Regardless of how you come by your answer, you must write up the answer on your own.
Your proofs should not only be rigorous and free of error but also clear enough that I (or you) could use them to teach other students this material. You need to learn how much detail to include in a proof to make your argument clear. This is a judgment call (and it also depends on the audience), but it is critical.

There will be two tests. I will give you a list of results whose proofs you must know for the exams. Each exam will consist of five problems: three or four proofs selected from the list, plus one or two more novel questions.

Tentatively, the first test will be in class on Monday, November 6. The second test will be on the scheduled test date, December 20, 1-3pm.

I take improvement into account: if you flunk the first test but ace the second you ace the course; if you ace the first test but flunk the second, your final grade will not be good. I award grades of A and A+ only to students who consistently hand in homework on time. Otherwise, homework will not factor directly into your grade (but homework performance tends to be highly correlated with test performance).

I will not offer a makeup for either test. To be excused, you must give notification before the exam begins either to me or to the graduate secretary, Carissa Re. If you are excused, your course grade will then depend entirely on your performance on the other test.

3 Texts

Rudin (1976) is the official text. But I rely primarily on my own notes, most of which are in good shape and which are posted here. Rudin (1976) is extremely terse but it is an excellent technical reference and you should have some book of this sort in your library. Although there are many other fine analysis texts, and although Rudin (1976) is now decades old, it remains first among equals.

Below, I also provide an annotated bibliography of other relevant texts. You should look at it only as the need arises, which probably won’t be until after your prelims, if then.

4 Course Outline

4.1 Analysis Core

1. Logic. My discussion of mathematical logic will be brief. The important things to retain are the truth table interpretations of the sentential connectives (you should know what “imply” means), a facility with truth tables generally, an intuitive understanding as to why proofs by contraposition and reductio ad absurdum are valid, and an understanding of what axioms are.

2. Set Theory. My discussion of set theory will be brief. I survey set theory as an example of an axiomatic theory. I discuss in detail only a few topics in set
theory, notably the construction of the natural numbers, induction, functions, and cardinality.

3. The Real Field. The real field, $\mathbb{R}$, is the set of real numbers together with the basic arithmetic operations, addition and multiplication. Analysis makes heavy use of the fact that the $\mathbb{R}$ exhibit the least upper bound property: any set of reals that is bounded above has a least upper bound. This property is not exhibited by the rational numbers.

4. Normed Vector Spaces. A vector space is a set $X$ together with operations called vector addition and scalar multiplication that must satisfy certain properties. Any vector space has a special element called the origin. A norm on a metric space measures distance from the origin. Typically, many different norms are possible for the same vector space. Most spaces used in economics are vector spaces or nice subsets of vector spaces.

The canonical example of a normed vector space is $\mathbb{R}^N$, together with the Euclidean norm, the standard measure of distance from the origin in $\mathbb{R}^N$. An element of $\mathbb{R}^N$ has the form $(x_1, \ldots, x_N)$, where each $x_n$ is an element of $\mathbb{R}$. $\mathbb{R}^N$ is also called $N$-dimensional Euclidean space.

5. Metric Spaces. Given a set of points $X$ a metric on $X$ measures the distance between any two elements of $X$. If $X$ is a normed vector space, the norm induces a metric in a natural way. In particular, in $\mathbb{R}^N$, the standard metric, called the Euclidean metric, is based on the Euclidean norm. But a general metric space need not be a vector space and, more importantly for our purposes, even if it is, we can use metrics that are not based on any norm. We will encounter an important example of this in the context of $\mathbb{R}^\omega$ (an infinite-dimensional analog of $\mathbb{R}^N$).

A metric is used to define what it means for a set to be open. Openness is the fundamental concept of topology and is also vital to analysis. Openness is used in turn to define other topological properties such as closedness, compactness, and connectedness.

6. Compactness. Compactness is a critical property for many economic applications. In metric spaces, compactness holds if and only if the set is complete and totally bounded. Completeness is a metric space property that implies a weaker property called closedness. Total boundedness is a metric space property that implies a weaker property called boundedness. The Heine-Borel Theorem states that, in $\mathbb{R}^N$ it suffices to check closedness and boundedness: in $\mathbb{R}^N$, any closed and bounded set is compact. This fact is convenient because it is often very easy to check closedness and boundedness. Unfortunately, I show that in $\mathbb{R}^\omega$ and infinite-dimensional subspaces thereof, with various metrics, closed and bounded sets need not be compact.
7. **Continuous Functions.** The mathematical field of topology is, among other things, concerned with what topological properties of sets are preserved by continuous transformation. Both compactness and connectedness are preserved by continuous transformation.

The topological invariance of compact sets implies the existence of a solution in many optimization problems, a result of obvious importance to economics.

8. **Basic Measure Theory.** My goal is modest: to get you to understand what Lebesgue measure is, why we have to worry about “measurability,” and to provide an introduction to Lebesgue integration.

### 4.2 Modules

Over the years, I have developed a number of modules. This, year I plan to cover the following, although this list is subject to change and, because of time constraints, I may not get to some topics (e.g., proving the Tarski Fixed Point Theorem).

1. **Convex Sets and Basic Separation Theorems.** The main goal is to prove the basic separation and support theorems for convex sets in $\mathbb{R}^N$. These results underly a number of important results in economics: (a) in optimization, the Karush-Kuhn-Tucker Theorem (KKT) and the closely related result on saddle points, (b) in General Equilibrium Theory, the Second Welfare Theorem (any efficient allocation can be interpreted as a competitive equilibrium allocation), (c) in Finance, the Fundamental Theorem of Asset Pricing (asset prices have a natural interpretation as present values), (d) and in Game Theory, the equivalence between strict dominance and never a best response. In section, I will prove a version of KKT using a support theorem for closed pointed cones.

2. **Fixed Point Theorems.** Fixed point theorems are widely used to prove existence of equilibrium and also show up in other contexts (e.g., the Contraction Mapping Theorem is used in Dynamic Programming). I prove the Brouwer Fixed Point Theorem and sketch a proof of the Kakutani Fixed Point Theorem. I may prove the Tarski Fixed Point Theorem. I survey some other fixed point theorems.

3. **Monotone Comparative Statics.** Monotone Comparative Statics provides tools for making comparative statics statements (an increase in this causes an increase in that) with weak assumptions and without using calculus.

Other possible modules include Vector Spaces and Dynamical Systems.

### 5 More Texts

I am including the following texts by way of reference. I do not expect that you will look any of them up this semester.
5.1 Mathematics Texts

In addition to Rudin (1976), there are a number of other basic analysis texts, including Abbott (2002), Apostol (1974), Dieudonné (1969), and Krantz (1991).

After basic analysis, the next step up is to measure theory and functional analysis, often folded into a single course sequence with a title like “real variables.” Standard texts for this include Kolmogorov and Fomin (1970), Halmos (1978), and Royden (2010). And measure theory is covered in advanced probability texts, such as Billingsley (1995). For functional analysis, a good resource is Aliprantis and Border (1999), which is written expressly for economists.

Analysis and topology are filled with intuition confounding examples. Gelbaum and Olmstead (2003) and Steen and Seebach, Jr. (1995) collect many of these.

A good introductory reference for classical mathematical logic is Enderton (2000). But with one major exception, economics rarely makes explicit use of mathematical logic. The major exception is that in recent decades there has been interest among decision theorists and game theorists in special types of formal logics designed to capture aspects of reasoning about knowledge. An informal introduction can be found in Rubinstein (1998).

For set theory, Halmos (1970) is the classic introductory text. It provides most of the set theory that is actually used in mathematics (and in economics) without getting bogged down in formalism. If you find that you crave a more formal treatment of set theory, then I recommend Enderton (1977). An even more formal treatment can be found in Suppes (1960).

My favorite introduction to general topology is Munkres (1999). An older book, Kelly (1955), may also prove useful. Some parts of economic theory have exploited the machinery of differential topology (which is topology using differentiable, rather than merely continuous, functions). The classic introduction is Milnor (1997).

On convexity, the classic reference is Rockafellar (1970). It is also a classic cite for finite dimensional (convex) optimization. But you may find it difficult.

For optimization, both finite and infinite dimensional, the classic (but advanced) text is Luenberger (1969).

For a nice treatment of fixed point theory, see Border (1985), which was written by an economist for economists.

5.2 Math for economists texts.

There are a number of analysis for economists texts, written by economists for courses like this one, have appeared. These include Corbae, Stinchcombe and Zeman (2009), Ok (2007), Carter (2001), and de la Fuente (2000). In addition, there are online notes for courses more or less like this one. Chris Shannon (Berkeley) has a nice set of notes, available here. And Leo Simon (also Berkeley) has nice notes here.

These texts are in addition to the more traditional, optimization centric, math for economists texts, such as Simon and Blume (1994), Sundaram (1996), and Novshek
Intriligator (2002) is older (it is a reprinting of a 1971 book) and more elementary but I find it useful. Takayama (1985) is an encyclopedic tour of a very large number of topics in classical mathematical economics.


The standard text for monotone comparative statics is Topkis (1998).

6 Help.

There will be an extra weekly section on Fridays, 1-2:30 PM, in Seigle 106 (we may not use the whole time period). The first of these will be September 2. Some of the sections will be devoted to going over problems, clarifying material from class, and that sort of thing. Some of the sections, however, will discuss new material. I will hold some of the latter sections; the assistant to the instructor will hold the others. I will not test you on any new material introduced in section.

My office hours are Tuesdays, 2pm-4pm, or by appointment. e-mail is the best way to reach me (I hate using the phone, to be frank).

Office: Seigle 386
e-mail: nachbar@wustl.edu

The assistant to the instructor for this course is still to be determined.

7 Information from the Provost’s office.

Accommodations based upon sexual assault: The University is committed to offering reasonable academic accommodations to students who are victims of sexual assault. Students are eligible for accommodation regardless of whether they seek criminal or disciplinary action. Depending on the specific nature of the allegation, such measures may include but are not limited to: implementation of a no-contact order, course/classroom assignment changes, and other academic support services and accommodations. If you need to request such accommodations, please direct your request to Kim Webb, kim.webb@wustl.edu, Director of the Relationship and Sexual Violence Prevention Center. Ms. Webb is a confidential resource; however, requests for accommodations will be shared with the appropriate University administration and faculty. The University will maintain as confidential any accommodations or protective measures provided to an individual student so long as it does not impair the ability to provide such measures.
Bias Reporting: The University has a process through which students, faculty, staff and community members who have experienced or witnessed incidents of bias, prejudice or discrimination against a student can report their experiences to the University’s Bias Report and Support System (BRSS) team. See brss.wustl.edu

Mental Health: Mental Health Services’ professional staff members work with students to resolve personal and interpersonal difficulties, many of which can affect the academic experience. These include conflicts with or worry about friends or family, concerns about eating or drinking patterns, and feelings of anxiety and depression. See: shs.wustl.edu/MentalHealth.

References


